Name:__________________________

SM3 2.3: Polynomial Long Division

<u>Vocabulary</u>: Polynomial, Descending Order, Dividend, Divisor, Quotient, Remainder

Divide $2654 \div 3$

Remember the game you played in grade school called long division? Let's revisit the rules of the game.

Set up $3\overline{)2654}$

We set the number to be divided (dividend) within a division container.

The value of the expression (quotient) will appear above the container as we complete the process.

The number doing the dividing (divisor) is set left of the container.

Process $3\overline{)2654}$

Divide as few digits of the dividend, starting with the left digit, that will are at least as large as the divisor.

In this example, 2 is smaller than 3, so we have to use the second digit and consider the division of 26 by 3.

$26 \div 3 = 8$, with a remainder of 2.

$$\begin{array}{r} 8 \\ 3\overline{)2654} \\ -\underline{24} \\ 2 \end{array}$$

Write the first term of the quotient that comes from the first division done. Then multiply that term by the divisor and write the product beneath the portion of the dividend used. Subtract the product from the used dividend and write the difference underneath.

$$\begin{array}{r} 8 \\ 3\overline{)2654} \\ -\underline{24}\downarrow \\ 25 \end{array}$$

Bring down the next digit of the dividend and repeat the process using the lower number as the new portion of dividend.

$$\begin{array}{r} 88 \\ 3\overline{)2654} \\ -\underline{24} \\ 25 \\ -\underline{24}\downarrow \\ 14 \end{array}$$

Continue this process until there is no dividend left to pull down.

$$\begin{array}{r} 884\ {}^{2}/_{3} \\ 3\overline{)2654} \\ -\underline{24} \\ 25 \\ -\underline{24} \\ 14 \\ -\underline{12} \\ 2 \end{array}$$

Once out of digits of the dividend to bring down, the final number at the bottom is the undivided portion of the original dividend. It has yet to be divided by the divisor, and is called the remainder. We write the remainder as the numerator of a fraction with the divisor as the denominator and add that to the rest of the quotient.

Polynomial Long Division follows the same rules as number long division, but with the division of two polynomials. We use terms of polynomials in the same manner that we used digits of numbers.

Divide $(x^2 - 5x + 1) \div (x - 2)$

Set up

$$x - 2 \overline{\big)\, x^2 - 5x + 1}$$

The process for preparing to divide is the same: dividend inside the container, divisor to the left, the quotient will appear above. The polynomials should be written in descending order.

Process

$$x - 2 \overline{\big)\, x^2 - 5x + 1}$$

The control of the division is given to the most powerful term of the divisor. In this example, that term is x.

We'll also only use the first term of the dividend. In this example, the portion of the dividend we'll use is x^2.

$$\begin{array}{r l} & x \\ x - 2 & \overline{\big)\, x^2 - 5x + 1} \\ & \underline{-(x^2 - 2x)} \\ & \quad -3x \end{array}$$

Divide: $x^2 \div x = x$

Write the first term of the quotient that comes from the first division done, x. Then multiply that term by the divisor, $x - 2$, and write the product, $x^2 - 2x$, beneath the portion of the dividend used. Subtract the product from the used dividend and write the difference underneath. Be careful of sign errors!

$$\begin{array}{r l} & x \\ x - 2 & \overline{\big)\, x^2 - 5x + 1} \\ & \underline{-(x^2 - 2x)} \downarrow \\ & \quad -3x + 1 \end{array}$$

Bring down the next term of the dividend and repeat the first step using the lower number as the new portion of dividend.

$$\begin{array}{r l} & x - 3 \\ x - 2 & \overline{\big)\, x^2 - 5x + 1} \\ & \underline{-(x^2 - 2x)} \\ & \quad -3x + 1 \\ & \quad \underline{-(-3x + 6)} \\ & \quad\quad -5 \end{array}$$

Repeat until the dividend is consumed by the process. The polynomial at the bottom that has not been divided is the remainder.

$$\begin{array}{r l} & x - 3 + \dfrac{-5}{x - 2} \\ x - 2 & \overline{\big)\, x^2 - 5x + 1} \\ & \underline{-(x^2 - 2x)} \\ & \quad -3x + 1 \\ & \quad \underline{-(-3x + 6)} \\ & \quad\quad -5 \end{array}$$

We write the remainder as the numerator of a fraction with the divisor as the denominator and add that as part of the quotient.

Warning: Numbers do not forget to mention their digits. 705 has a 0 in the tens place. Polynomials only bring nonzero terms to your attention. Use placeholders to remain uniform! So, you'd write $x^3 + x - 5$ as $x^3 + 0x^2 + x - 5$ so that the process works.

Vocabulary Problems: Answer the questions about each function:

1) $a(x) = \frac{4x^2+2x+3}{x-8}$

The dividend of $a(x)$ is:

The divisor of $a(x)$ is:

2)

$$\begin{array}{r|l} & x+4 \\ \hline x-1 & x^2+3x+7 \\ & -(x^2 \quad -x) \\ \hline & \qquad 4x+7 \\ & \qquad -(4x-4) \\ \hline & \qquad\qquad 11 \end{array}$$

The dividend is:
The divisor is:
The remainder is:
The $\frac{remainder}{divisor}$ is:
The quotient is:

Setting up: Set up the long divisions with appropriate placeholders, but do not find the quotients:

3) $\frac{75x^4 - 3}{x^2 - 4x - 8}$

4) $\frac{2 - 5x + x^3}{1 - 3x^2}$

Problems: Find each quotient.

5) $\frac{x^2 + 10x + 24}{x + 6}$

6) $\frac{x^2 - 25}{x - 5}$

7) $\frac{5x^2 + 8x + 7}{x + 4}$

8) $\frac{x^3 + 4x^2 + 9x + 36}{x + 4}$

9) $\frac{2x^3 - 2x^2 - x - 3}{x - 1}$

10) $\frac{x^3 + 5x}{x^2 - 2}$

11) $(x^2+5x-7)\div(x-2)$

12) $(x^2+7)(x+3)^{-1}$

13) $(x^3-x^2+x-2)(x^2+3x-1)^{-1}$

14) $\dfrac{x^4-1}{x-1}$

15) $\dfrac{2x-5}{x^2-3x-7}$

Composite Problems: Find each quotient.

16) $\dfrac{(x^2+4)+(2x^2+x-5)}{x+10}$

17) $\dfrac{(2x-1)^3}{x-2}$

18) $\dfrac{(x-3)(x^2-x-6)}{x+2}$

19) $\dfrac{(x+5)^3}{x-1}$

Application: Use polynomial long division to find the quotient.

20) The volume of a classroom is given by $v(x)=x^3-9x^2+23x-15\ ft^3$. The surface area of the floor of the classroom is given by $x^2-4x+3\ ft^2$. Assuming the classroom is a rectangular solid, find an expression for the height, in ft of the classroom.